

PC11: Unit 4 Practice Short Answer Questions

1. A baking company claims that their cookies contain "40 % chocolate chips by mass". To uphold their claim, what mass of chocolate chips should they add to 14 kg of cookie dough? Give the answer to the nearest tenth where necessary.

Let x = amount of Chocolate chips

$$\frac{40}{100} = \frac{x}{14+x}$$

$$\therefore x \approx 9.3 \text{ kg}$$

They would need to add $\approx 9.3 \text{ kg}$ of Chocolate Chips

$$\begin{aligned} 40(x+14) &= 100x \\ 40x + 560 &= 100x \\ 560 &= 60x \end{aligned}$$

2. Karen and Benjamin are ecologists studying forest regeneration. Working together, they can count the seedlings on a research site in 2 h. It takes Karen 8 h to count the seedlings on her own. How long would it take Benjamin to count the seedlings on his own? Give the answer to 1 decimal place where necessary.

Let x = Benjamin's time

$$\frac{2}{8} + \frac{2}{x} = 1$$

$$\text{CD: } 8x$$

It would take Benjamin 2.7 hours to count on his own.

$$\begin{aligned} 2x + 16 &= 8x \\ 16 &= 6x \\ x &= \frac{8}{3} \approx 2.7 \text{ hours} \end{aligned}$$

3. Dayna plants a tomato seed and a sunflower seed for a science project. She finds that on average her sunflower plant grows three times as fast as her tomato plant. It takes the tomato 20 days longer to reach a height of 14 cm. What is the growth rate, in centimetres per day, of the tomato seedling? Give the answer to the nearest hundredth where necessary.

Let x = days growing

$$\frac{14}{x+20} = \frac{14}{3x}$$

$$14(3x) = 14(x+20)$$

$$42x = 14x + 280$$

$$28x = 280$$

$$\therefore x = 10$$

It would take the tomatoes 0.47 cm/day of growth.

$$\text{Tomato Speed} = \frac{14}{(10)+20} = \frac{14}{30} = 0.47 \text{ cm/day}$$

4. A freight train travels 60 km. A single locomotive pulls the train for the first half of the trip, then a second locomotive is added, doubling the speed of the train. If the total time for the trip is 54 min, what is the speed of the train with one locomotive?

Handwritten solution for problem 4:

60 km = ^{total} distance
 54 min = ^{total} time
 ($\frac{54}{60}$ hrs).

1 locomotive - 30 km (speed = X km/h)
 2 locomotives - 30 km (speed = 2X km/h)

LCM: 60x.

$$\frac{30}{x} + \frac{30}{2x} = \frac{54}{60}$$

$$\frac{1800}{60x} + \frac{900}{60x} = \frac{54x}{60x}$$

$$\frac{2700}{54} = \frac{54x}{54}$$

$$x = 50$$

∴ The speed of the train with one locomotive is 50 km/h.

5. A liquid fertilizer requires dilution before use. How much liquid fertilizer must be added to 600 mL of water to make a 25% fertilizer solution?

Let x = amount of fertilizer added

CD: $100(x+600)$

$$\frac{25}{100} = \frac{x}{600 + x}$$

$$25(600 + x) = 100x$$

$$25x + 15000 = 100x$$

$$15000 = 75x$$

$$x = 200 \text{ mL}$$

∴ 200mL of Fertilizer must be added

6. Leah and Rain travel 2 km to the aquatic centre. Leah rides her bike while Rain rides an electric scooter. Leah's average speed is 1 km/h greater than Rain's. Leah arrives at the centre 10 min before Rain. What is Rain's average speed on her scooter?

Let x = Rain's Speed

CD: $6x(x+1)$

$$\frac{2}{x} + \frac{10}{60} = \frac{2}{x+1}$$

\therefore Rain's average speed is 3km/hr

$$12(x+1) + x(x+1) = 12x$$

$$12x + 12 + x^2 + x = 12x$$

$$x^2 + x + 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = 3 \text{ OR } x = -4$$

$$\therefore x = 3 \text{ km/hr}$$

7. Theo and Stefan are apprentice auto mechanics. Together, they can change the oil and filter in a car in 20 min. Working alone, Theo can change the oil and filter in 36 min. How long would it take Stefan to change the oil and filter on his own?

Let x = Stefan's time to change the oil

CD: $36x$

$$\frac{20}{36} + \frac{20}{x} = 1$$

$$20x + 36(20) = 36x$$

$$20x + 720 = 36x$$

$$720 = 16x$$

$$x = 45 \text{ minutes}$$

\therefore it takes Stefan 45 minutes to change the oil on his own

8. A natural number, N , is 4 less than another natural number, M . The sum of the reciprocals of M and N is 5 times the reciprocal of twice the value of M . What are the two numbers? Describe your strategy.

N is 4 less than M , so $N = M - 4$.

The sum of the reciprocals of M and N is: $\frac{1}{M} + \frac{1}{M-4}$

Five times the reciprocal of twice the value of M is: $\frac{5}{2M}$

An equation for the two numbers is: $\frac{1}{M} + \frac{1}{M-4} = \frac{5}{2M}$

$M = 0$ and $M = 4$ are non-permissible values.

Solve the equation. A common denominator is: $2M(M-4)$

Multiply all terms in the equation by the common denominator and simplify.

$$2M(M-4)\left(\frac{1}{M}\right) + 2M(M-4)\left(\frac{1}{M-4}\right) = 2M(M-4)\left(\frac{5}{2M}\right)$$

$$2(M-4) + 2M = 5(M-4)$$

$$2M - 8 + 2M = 5M - 20$$

$$4M - 8 = 5M - 20$$

$$-8 + 20 = 5M - 4M$$

$$M = 12$$

$$N = M - 4$$

$$N = 12 - 4$$

$$N = 8$$

The larger number M is 12 and the lesser number N is 8.

9. A cyclist rode from town A to town B and back, a distance of about 3 km each way. On the trip out, there was an 8 km/h tailwind. On the return trip, there was a 6 km/h headwind. The total riding time was 5 h. To the nearest tenth of a kilometer per hour, what is the cyclist's average speed when there is no wind? Explain your solution.

Let x km/h represent the average speed of the cyclist when there is no wind.

An equation representing the total riding time is:

$$\frac{3}{x+8} + \frac{3}{x-6} = 5, x > 6$$

Solve the equation. A common denominator is: $(x+8)(x-6)$

Multiply all terms in the equation by the common denominator and simplify.

$$\frac{3}{x+8} + \frac{3}{x-6} = 5$$

$$(x+8)(x-6)\left(\frac{3}{x+8}\right) + (x+8)(x-6)\left(\frac{3}{x-6}\right) = 5(x+8)(x-6)$$

$$3(x-6) + 3(x+8) = 5(x-6)(x+8)$$

$$5x^2 + 4x - 246 = 0$$

Solve the equation $5x^2 + 4x - 246 = 0$ using the quadratic formula.

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(5)(-246)}}{2(5)}$$

$$x = \frac{-4 \pm \sqrt{4936}}{10}$$

$$x \doteq 6.6 \text{ or } x \doteq -7.4$$

Since speed cannot be negative, $x \doteq 6.6$.

The cyclist's average speed when there is no wind is approximately 6.6 km/h.